

# Fermionic light in common optical media

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Recent experiments have proven that the response to short laser pulses of common optical media, such as air or Oxygen, can be described by focusing Kerr and higher order nonlinearities of alternating signs. Such media support the propagation of steady solitary waves. We argue by both numerical and analytical computations that the low power fundamental bright solitons satisfy an equation of state which is similar to that of a degenerate gas of fermions at zero temperature. Considering in particular the propagation in both  $O_2$  and air, we also find that the high power solutions behave like droplets of ordinary liquids. We then show how a grid of the fermionic light bubbles can be generated and forced to merge in a liquid droplet. This leads us to propose a set of experiments aimed at the production of both the fermionic and liquid phases of light, and at the demonstration of the transition from the former to the latter.

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In suitable optical media, light has been argued to acquire material properties. A long dated example is the equivalence of the paraxial propagation of a laser pulse in a Kerr medium with the time evolution of a superfluid Bose-Einstein condensate, due to the identity of the nonlinear Schrödinger equation with the Gross-Pitaevskii equation[1]. More recently, optical induction has been used to create photonic crystals[2]; a photonic system has been designed that may undergo a Mott insulator to superfluid quantum phase transition[3]; soliton solutions for light propagation in Cubic-Quintic (CQ) nonlinear media have been shown to behave like ordinary liquids[4, 5].

On the other hand, recent experimental and theoretical works have proven that the response to ultrashort laser pulses of common optical media, such as air or Oxygen, can be described by focusing Kerr[6] and higher order nonlinearities of alternating signs[7], which have also been argued to provide the main mechanism in filament stabilization, instead of the plasma defocusing[7].

In this letter, we demonstrate by analytical and numerical computations that such media can support the propagation of steady solitary waves that appear in two clearly different phases. The low power solitons are governed by the same equation of state than a degenerate gas of fermions. We will call such a system “*Fermionic Light*”. On the other hand, the high power localized states satisfy the Young-Laplace equation that governs the formation of droplets in ordinary liquids, similarly to the result that was recently obtained for the CQ model[5]. We also show how to generate a grid of fermionic light bubbles and make it turn into a liquid light droplet.

We will consider the (paraxial) propagation along the  $z$ -direction of a linearly polarized laser beam, so that the complex electric field component  $\Psi(x, y, z)$  satisfies the nonlinear Schrödinger equation

$$i \frac{\partial \Psi}{\partial z} + \frac{1}{2k_0 n_0} \nabla_{\perp}^2 \Psi + k_0 \Delta n \Psi = 0, \quad (1)$$

where  $n_0$  is the linear refractive index of the medium,

$\nabla_{\perp}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$  is the transverse Laplace operator, and  $k_0 = 2\pi/\lambda_0$  is the mean wavenumber in vacuum, where  $\lambda_0$  is the central wavelength of the laser source which will be fixed to  $\lambda_0 = 800nm$  throughout this paper as in the experiment of Ref.[7]. For the optical media that have been studied in Ref.[7], the nonlinear correction  $\Delta n$  to the refractive index can be expanded as

$$\Delta n \simeq n_2 |\Psi|^2 + n_4 |\Psi|^4 + n_6 |\Psi|^6 + n_8 |\Psi|^8, \quad (2)$$

with alternating sign coefficients  $n_2, n_6 > 0$  and  $n_4, n_8 < 0$ , that contribute to focusing and defocusing respectively. Taking into account that the values of the second-order dispersion and multiphoton-absorption coefficients for the air are  $k'' = 0.2 \text{ fs}^2/\text{cm}$  and  $\beta = 1.27 \times 10^{-126} \text{ cm}^{17}/W^9$ [8], respectively, we have checked that both effects do not lead to significant corrections in our results. To be concrete, we will perform most of the numerical calculations in the case of  $O_2$  as the propagation medium, taking the mean values obtained in the experiment[7],  $n_2 = 1.6 \times 10^{-19} \text{ cm}^2/W$ ,  $n_4 = -5.2 \times 10^{-33} \text{ cm}^4/W^2$ ,  $n_6 = 4.8 \times 10^{-46} \text{ cm}^6/W^3$ ,  $n_8 = -2.1 \times 10^{-59} \text{ cm}^8/W^4$ . For comparison, however, we will also mention the results that we obtain for air, using the corresponding values for the coefficients  $n_{2q}$  that are also given in Ref.[7].

We will search for finite localized solutions of Eq. (1) of the form  $\Psi(x, y, z) = \Phi(r)e^{-i\mu z}$ , where  $r = \sqrt{x^2 + y^2}$  and  $\mu$  is the propagation constant. Fig. 1 shows the result of our numerical computation for  $\Phi(0)$  in the existence domain of such solitons in Oxygen ( $\mu_{\infty} < \mu < 0$ ), where  $\mu_{\infty} = -0.096 \text{ cm}^{-1}$ , and for the radial profiles  $\Phi(r)$  of three of them (see left inset in Fig.1), corresponding to the low power (black solid line), moderate power (red dashed line) and high power regimes (blue dashed-dotted line) respectively. We express the transverse spatial variables  $x, y$  in terms of the adimensional coordinates  $\xi, \chi$ , measured in units of  $(n_4/n_0)^{1/2}(k_0 n_2)^{-1}$ . The amplitude  $|\Phi(0)|$  is measured in units of  $(n_2/n_4)^{1/2}$ .

Like in the CQ case,  $\mu$  can be identified with the chem-

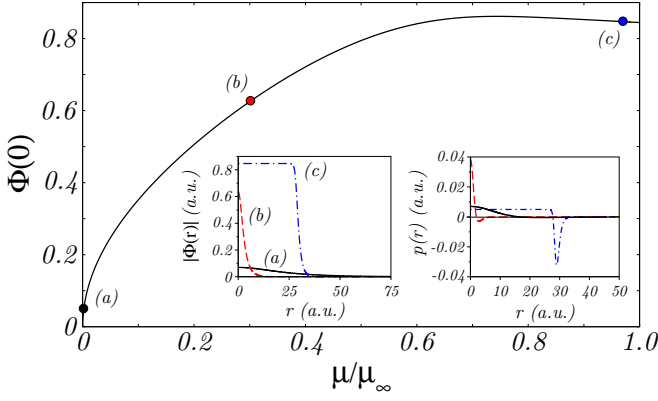


FIG. 1: [Color online] Plot of the central amplitude  $\Phi(0)$  vs.  $\mu/\mu_\infty$  for the family of fundamental eigenstates of Eq.(1). Note that  $\Phi(0)$  has a maximum for  $\mu/\mu_\infty \approx 0.7$ , due to the defocusing nonlinearities. Left inset: radial profiles of three eigenfunctions of Eq.(1) with  $\mu/\mu_\infty = 0.004$  (black solid),  $\mu/\mu_\infty = 0.3$  (red dashed) and  $\mu/\mu_\infty = 0.97$  (blue dashed-dotted) respectively. Right inset: radial pressure profiles corresponding to these solutions. The low-power profile is magnified by a factor  $10^3$  for clarity. Labeled points on the main curve refer to the eigenstates displayed within the insets.

ical potential of an equivalent thermodynamical two-dimensional system[5] of  $N = \int \rho dx dy \equiv \int |\Phi|^2 dx dy$  particles, described by the Landau's grand potential[10]  $\Omega = - \int p dx dy$ , where the pressure field  $p$  is now

$$p = -\frac{1}{2k_0 n_0} |\nabla_\perp \Phi|^2 + \mu |\Phi|^2 + k_0 \sum_{q=1}^4 n_{2q} \frac{\Phi^{2(q+1)}}{q+1} \quad (3)$$

For our optical system,  $\mu$ ,  $N$ ,  $\Omega$  and  $p$  correspond to the propagation constant, the power, the Lagrangian leading to Eq. (1), and the Lagrangian density, respectively.

The right inset of Fig.1 shows the pressure distributions  $p(r)$ , measured in units of  $k_0 n_2^3 n_4^{-2}$ , corresponding to the stationary states discussed above. Notice that all the radial distributions display both positive and negative pressure regions, being this a necessary condition for the existence of solitary waves. For the low power soliton (black solid line), corresponding to small values of  $|\mu|$ , we can obtain an analytical expression by using the variational method with the ansatz  $\Phi(A, r) = A \exp(-r^2/R^2)$ . The values of  $A(\mu)$  and  $R(\mu)$  that minimize  $\Omega$  for a given value of  $\mu$  can then be used to compute the pressure distribution. Taking the first non-vanishing order in  $|\mu|$  and inverting the dependence  $R(\mu)$ , we find a simple analytical approximation for the central pressure  $p_c$  as a function either of the radius  $R = \sqrt{2\langle r^2 \rangle}$  of the soliton, or of the central density  $\rho_c = |\Phi(0)|^2$  (corresponding to the beam intensity),

$$p_c = \frac{a}{k_0^3 n_0^2 n_2} \frac{1}{R^4} = b k_0 n_2 \rho_c^2, \quad (4)$$

with  $a = 4$  and  $b = 1/4$ . This relation is similar to the equation of state of a degenerate gas of fermions of mass

$m$  at zero temperature. In fact, if we apply the general definition of the Fermi momentum[10]  $P_F$  to a two dimensional system, we obtain  $P_F = \hbar \sqrt{2\pi\rho}$ , with  $\rho$  the density of the Fermi gas. As a consequence, the pressure, defined as the average force on a unit orthogonal line in the gas, can be obtained from the average kinetic energy as follows

$$p = \rho \langle E_{kin} \rangle = \frac{\rho}{2m} \frac{\int_0^{P_F} P^2 P dP}{\int_0^{P_F} P dP} = \frac{\pi \hbar^2}{2m} \rho^2 \quad (5)$$

which shows the same dependence with  $\rho^2$  as Eq. (4). This proves the formal analogy of our low power solitons with a degenerate Fermi gas in the central region around  $r = 0$ , which is arbitrarily large in the limit  $\mu \rightarrow 0$  (corresponding to large radius  $R \rightarrow \infty$ ). For these reasons, we will call 'Fermionic' the phase where the pressure is proportional to  $\rho^2$ .

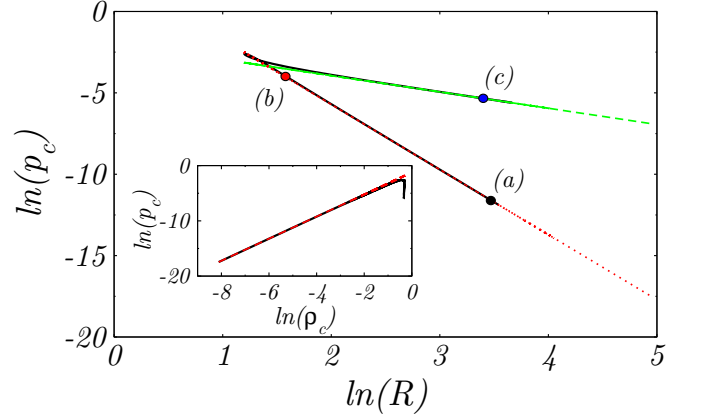


FIG. 2: [Color online] Plot of the logarithms of  $p_c$  vs.  $R$  for the fundamental solitons (black line). The fermionic behavior of Eq.(4) (red dotted) and the liquid YL equation (green dashed) are compared with the numerics. The labeled points correspond to the same eigenstates displayed in Fig.1. For each value of  $R$ , two different outcomes are possible depending on the power, corresponding to two different phases of light. Inset: plot of  $\ln(p_c)$  vs.  $\ln(\rho_c)$  (black solid) overlapped with Eq.(4) (red dashed) for comparison.

Note that in the limit  $\mu \rightarrow 0$  our variational computation gives a constant  $N = \frac{2\pi}{k_0^2 n_0 n_2}$ , which is consistent with the known result for the power flow leading to the collapse threshold in a Kerr medium[9]. The magnitude of this power  $N$  lies in the range of few GW in both  $O_2$  and air, and can be interpreted as the threshold for the existence of the Fermionic Light solitons.

Fig.2 shows the numerical computation of  $p_c$  as a function either of  $R$  (black solid line) or  $\rho_c$  (inset: black solid line) for all the nodeless solitary states of the model. The lower branch, corresponding to the low power solitons, is in excellent agreement with the dependence described by Eq.(4), as it can be inferred from the fitting (red dotted) straight line with slope  $s_{low} = -4$ . Furthermore, in the

inset of Fig.2 we also show the quantitative agreement between theory and numerics by comparing  $p_c$  with  $\rho_c$  instead of  $R$ . In this case, the slope of the straight line is  $s_{inset} = 2$  thus demonstrating the quadratic dependence on  $\rho_c$  given by Eq.(4). However, the correct numerical values of the constants are  $a = 2.5$  and  $b = 0.29$ , in reasonable agreement with the result of the variational method given above. We have checked numerically that the asymptotic behavior represented by the red lines in Fig.2 is practically independent on the higher order nonlinearities  $n_{2q}$  ( $q > 1$ ), as predicted by the theory. In particular, these results can be directly applied to both Kerr and CQ models.

On the other hand, the high power localized solutions exhibit top-flat profiles with an inhomogeneous negative pressure profile on the border, similar to those of the solitons appearing in the CQ model[5]. As a consequence, the gradient term in Eq.(1) can be neglected close to the origin, and we get  $\mu = -k_0 \Delta n(0) = -k_0 \sum_{q=1}^4 n_{2q} \Phi(0)^{2q}$ . By generalizing the argument of Ref.[5] including the higher order nonlinearities, we obtain that these states obey the celebrated Young-Laplace (YL) equation[11],  $p_c = 2\sigma/R$ , describing the behavior of usual liquid droplets. The value of the surface tension is

$$\sigma = \frac{n_2}{\sqrt{2n_0 n_4}} \int_0^{\Phi_\infty} \left( -\mu_\infty \Phi^2 - k_0 \sum_{q=1}^4 n_{2q} \frac{\Phi^{2(q+1)}}{q+1} \right)^{1/2} d\Phi, \quad (6)$$

being  $\mu_\infty$  and  $\Phi_\infty$  the asymptotic values corresponding to the  $R \rightarrow \infty$  droplet, that can be computed by solving the equation  $p_c = 0$  (neglecting the Laplacian term). For the propagation in Oxygen, we have obtained the following variational estimations:  $\mu_\infty = -0.247(k_0 n_2^2 n_4^{-1}) = -0.096 \text{ cm}^{-1}$ ,  $\Phi_\infty^2 = 0.712(n_2 n_4^{-1}) = 2.19 \times 10^{13} \text{ W/cm}^2$ ,  $\sigma = 0.0715(n_2^2 n_4^{-3/2} n_0^{-1/2}) = 4.88 \times 10^9 \text{ W/cm}^2$ . These results, obtained assuming a top-flat function, are in excellent agreement with the computation given in Fig.1. In Fig.2, we show that our numerical solution in the case of  $O_2$  satisfy the YL equation (green dashed line) with very good accuracy for a wide range of values of  $\mu$ . On the other hand, for the propagation in air, the liquid light phase would correspond to a higher intensity,  $[\Phi_\infty^2]_a = 2.98 \times 10^{13} \text{ W/cm}^2$ , with  $[\mu_\infty]_a = -0.116 \text{ cm}^{-1}$  and  $[\sigma]_a = 7.16 \times 10^9 \text{ W/cm}^2$ , and the YL equation would still be valid.

As we have seen above, the propagation of self-guided light beams in media like Oxygen can occur in two clearly separated phases, satisfying two different equations of state. In fact, we have checked that qualitatively similar results can also be obtained for the CQ case, and occur whenever the nonlinear refractive index displays a single well-defined maximum as a function of the intensity. It would then be interesting to demonstrate the possibility of a transition between the fermionic bubbles

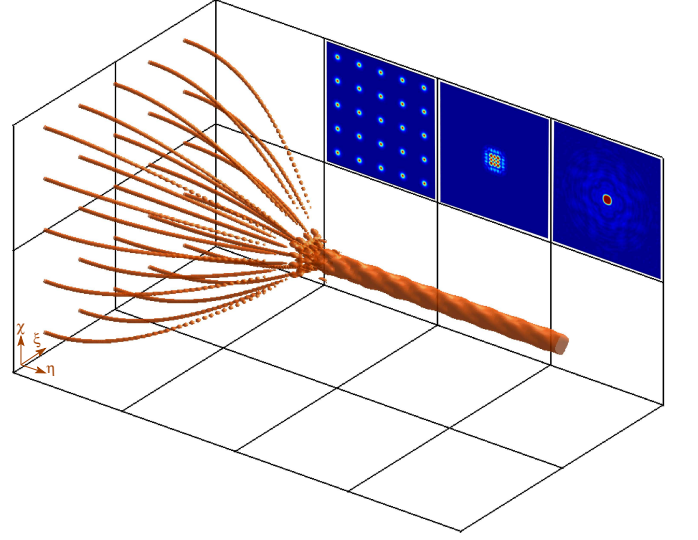


FIG. 3: [Color online] Isosurface intensity plot of the dynamical phase transition from a square grid of fermionic light solitons to an individual liquid light soliton in  $O_2$ . All solitons in the grid are eigenstates with  $\mu/\mu_\infty = 0.3$  displayed in Fig.1. They are forced to merge in the center of the computational window by means of an external harmonic potential. We can see three pseudocolor plots displaying the initial condition (left,  $\eta = 0$ ), the collapse of the light bubbles (middle,  $\eta = 200$ ) and the final state (right,  $\eta = 500$ ). In the latter we observe a flat-top soliton with radius  $R \approx 10$  and  $\rho_c \approx 0.76$  arising after the massive coalescence of the fermionic solitons. The width of the square spatial domain displayed is  $w_{\xi,\chi} = 200$ .

and the liquid droplets of light. A suggestive analogy is that of the collapse of a star, that occurs when the gravitational interaction overcomes the Fermi pressure of the electrons. We can obtain a qualitatively similar result in the case of light propagation in Oxygen, by compressing the fermionic bubbles using a harmonic potential, leading to the generation of a liquid light droplet. Fig. 3 shows the result of our simulation in  $O_2$ . The initial state (see left snapshot in Fig. 3) consists of a regular grid of fermionic light bubbles with  $\mu/\mu_\infty = 0.3$  (see their radial profile in Fig. 1), with a separation between nearest neighbors  $\Delta_{\xi,\chi} = 40$ . We include an external harmonic potential  $V(\xi, \chi) = \frac{K}{2}(\xi^2 + \chi^2)$  with  $K = 5 \times 10^{-5}$ , in Eq.(1) in order to induce a net force acting on the grid with the aim of making all the fermionic solitons to collide in the center of the computational window  $(\xi, \chi) = 0$ .

This parabolic potential can be obtained by inducing in the medium a "gas lens"[12], which can be constructed with an electrically heated pipe through which passes a laminar flow of gas. By controlling the differential heating at the boundaries and the velocity of the flow, a parabolic refractive index gradient can be obtained as documented in Ref.[12]. On the other hand, we have also checked numerically that the same qualitative result can be obtained by using a glass lens, provided that its focal

length  $f$  is at least ten times greater than the Rayleigh length of the input beams, which in the case depicted in Fig. 3 corresponds to  $f = 5$  m.

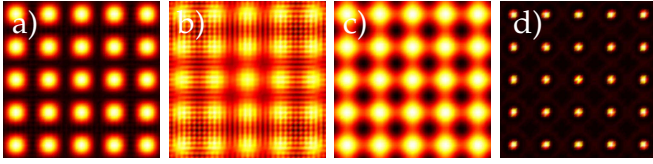


FIG. 4: [Color online] Proposal for the generation of the initial condition of Fig.3 in experimental setups. The (a) pseudo-color plot shows the bidimensional cosine squared-like phase distribution to be imprinted onto the ingoing beam. We assume that the beam is much wider than the filaments, and restrict our simulation to a square domain  $\xi, \chi \in [-75, 75]$  where the wavefront is assumed to be homogeneous. In snapshots (b)-(c) ( $\eta = 100, 200$ ) we observe how the beam profile starts to destabilize, leading to the formation of a grid of spatial solitons at  $\eta = 300$  (see snapshot (d)).

For simplicity, we have prepared the initial state with exact solutions of Eq.(1) to reduce the excitation of linear radiative modes. We have checked that such an initial guess can be generated experimentally by means of the modulational instability of a low power probe pulse. In fact, Fig.4 shows the results of a numerical simulation where we have added an initial cosine squared-type phase distribution (see left snapshot in Fig.4) to a homogeneous plane wave in order to excite a regular grid of solitons, as in Refs. [13]. As shown in snapshots b) to d), this allows us to control the spatial location of the optical filaments generated during the beam breakup due to modulational instability. The outgoing grid of spatial solitons is depicted in snapshot d). Note its similarity with the initial state of Fig.3.

Let us come back to the results of Fig.3. The massive coalescence of all the fermionic bubbles occurs after a finite propagation distance  $\eta = 200$  (see middle snapshot in Fig.3), measured in units of  $n_4 k_0^{-1} n_2^{-2}$ . As a result, a unique filament structure with large radius arises, as it can be appreciated in Fig.3. We have checked that this soliton is a flat-top eigenstate with radial perturbations coming from the transition process. In fact, we have estimated its logarithmic radius to be around 2.3 and its peak density  $\rho_c \approx 0.76$  (measured in units of  $n_2/n_4$ , thus corresponding with an intensity  $2.34 \times 10^{13} \text{ W/cm}^2$ ), which is clearly on the liquid light branch displayed in Fig.2. For Oxygen[7], we have considered a propagation distance of about 13 m. Although such a distance seems to be huge for a real experiment, we have verified that by stretching the external potential this distance can be realistically reduced by almost one order of magnitude. Finally, we note that the use of pressurized  $O_2$  or air may also help to enhance the nonlinear optical response. For all these reasons, we conclude that the demonstration of the existence of both Fermionic and Liquid light and the

phase transition between them would be an affordable challenge in real experiments.

In conclusion, we have proven that common media ( $O_2$ , air) can support the propagation of solitary waves that appear in two clearly different phases with unequal physical properties, namely the low power “Fermionic” light, satisfying an equation of state similar to that of a degenerate gas of fermions, and the high power “Liquid” light, obeying the YL equation. We have then shown how a grid of the fermionic light bubbles can be generated and forced to merge in a liquid droplet. We think that the possible experimental validation of our proposal could also provide an independent way to corroborate the deep change in the understanding of the filamentation process in gases that was proposed in Ref.[7]. Furthermore, these results in air pave the way for the improvement of recent experiments on laser-induced water condensation[14], built on top of these new robust light distributions.

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